A Spherical Pendulum System to Teach Key Concepts in Kinematics, Dynamics, Control, and Simulation

Fathi Ghorbel, *Member, IEEE,* and James B. Dabney

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I. Summary

The Rice Spherical PENDUlum Laboratory APparatus (SPENDULAP) is a rich teaching aid for senior and first-year graduate courses in kinematics, dynamics, control, and simulation. It consists of a free-swinging (unactuated) rigid pendulum mounted in a rotating frame. The frame rotates about an axis perpendicular to the pendulum swing axis and is driven by an electric motor. The SPENDULAP is attractive as a teaching tool because it is easy to visualize the motion of the pendulum, but somewhat challenging to model kinematically and dynamically. In particular, the three-dimensional nature of the pendulum motion allows students to gain proficiency and confidence not possible with a planar apparatus. Additionally, nonlinearities in the dynamics present interesting, but tenable, control challenges. In this paper, we illustrate each step in the process of kinematic and dynamic modeling, simulation, and control of the SPENDULAP. We start with the kinematic analysis, and then develop the equations of motion using both Newtonian and Lagrangian approaches. The spherical pendulum is sufficiently complex to demonstrate the advantages of the Lagrangian approach, and it also offers an excellent illustration of the benefits of the Newtonian formulation. Next, we illustrate the development of a numerical simulation of the SPENDULAP dynamics, and provide examples of uses of simulation and animation using MATLAB. Finally, we show the development of linear and nonlinear control laws, and illustrate testing them using simulation.

Fathi Ghorbel (S'86–M'87) received the B.S. degree with honors from the Pennsylvania State University, State College, in 1985, the M.S. degree from Carnegie-Mellon University, Pittsburgh, PA, in 1987, and the Ph.D. degree from the University of Illinois, Urbana-Champaign, in 1991, all in mechanical engineering.

He is an Associate Professor at the Department of Mechanical Engineering and the Department of Bioengineering at Rice University, Houston, TX, where he is also the Director of the Dynamic Systems and Control Laboratory, and the Robotics Laboratory, and the Codirector of the Biomedical Systems and Instrumentation Lab. His research is in the areas of systems and control theory, robotics, and biomedical engineering systems.

Dr. Ghorbel is a member of Sigma Xi, ASME, IFAC, SIAM, and IASTED. He is an Associate Editor for the IEEE Transactions on Control Systems Technology, an Associate Editor for the *International Journal of Robotics and Automation*, and an Associate Editor for the IEEE Control Systems Society Conference Editorial Board.

James B. Dabney received the B.S. degree in mechanical engineering from Virginia Polytechnic Institute and State University, Blacksburg, in 1974, the M.S. degree in process monitoring and control from University of Houston, Clear Lake, TX, in 1993, and the Ph.D. degree in mechanical engineering from Rice University, Houston, TX, in 1998.

He is a Lecturer and Postdoctoral Fellow in the Department of Mechanical Engineering at Rice University. His research includes the application of numerical optimization and optimal control techniques to problems in mechanical engineering, numerical simulation, and control.

Dr. Dabney is a member of AIAA and a reviewer for the *International Journal of Robotics and Automation*.

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The authors are with the Department of Mechanical Engineering, Rice University, Houston, TX, 77055 USA.
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